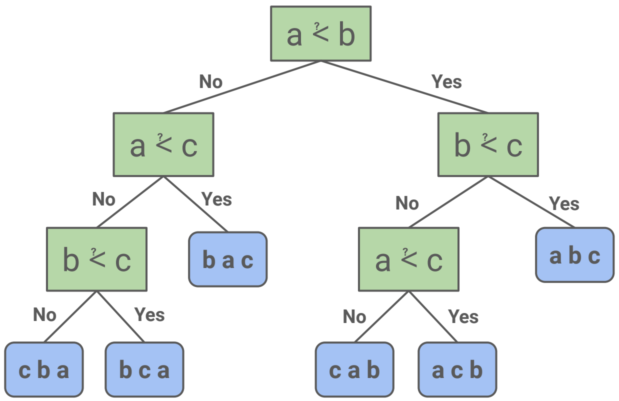


Median-of-3 Decision Tree


```
int[] decisionTreeSort(int a, int b, int c) {
  if (a < b) {
    if      (b < c) return {a, b, c};
    else if (a < c) return {a, c, b};
    else            return {c, a, b};
  } else {
    if      (c < b) return {b, a, c};
    else if (c < a) return {b, c, a};
    else            return {c, b, a};
  }
}
```

Sorting with Decision Trees

In the worst case, how many questions would you need to ask to definitively sort {a, b, c, d}?

- 3
- 4
- 5
- 6
- Not sure

Optimal Comparison Sort

Reductions set upper bounds on runtime.

Theoretical optimality sets lower bounds.

On random arrays, decision tree sorting is **optimal** in the number of comparisons.

Cost model. Number of comparisons.

Optimal decision tree sort doesn't exist. Provably optimal for N < 16 and N = 22.

Towards Optimal Sorting of 16 Elements

Marcin Peczarski

Institute of Informatics, University of Warsaw,
ul. Banacha 2, 02-097 Warszawa, Poland,
marpe@mimuw.edu.pl

Abstract. One of the fundamental problem in the theory of sorting is to find the pessimistic number of comparisons sufficient to sort a given number of elements. Currently 16 is the lowest number of elements for which we do not know the exact value. We know that 46 comparisons suffices and that 44 do not. There is an open question if 45 comparisons are sufficient. We present an attempt to resolve that problem by performing an exhaustive computer search. We also present an algorithm for counting linear extensions which substantially speeds up computations.

A036604: Sorting numbers: minimal number of comparisons needed to sort n elements (Neil Sloane/OEIS); Towards Optimal Sorting of 16 Elements (Marcin Peczarski/arXiv)

Upper Bound for N!

Goal. Find an asymptotic complexity bound for the function log(N!).

Subgoal. Find an **upper bound** for the function N!

$$N! = 1 \cdot 2 \cdot 3 \cdots (N-2) \cdot (N-1) \cdot N$$

Upper Bound for log(N!)

Goal. Find an asymptotic complexity bound for the function log(N!).

Subgoal. Find an **upper bound** for the function log(N!)

$$N! < N^N$$

Lower Bound for log(N!)

Goal. Find an asymptotic complexity bound for the function log(N!).

Subgoal. Find a **lower bound** for the function log(N!)

$$\begin{aligned} ? \leq \quad & N! \leq N^N \\ \log ? \leq \quad & \log N! \leq N \log N \end{aligned}$$

When poll is active, respond at **PollEv.com/kevinl**

What can we say about decision tree sorting?

- $\log N! \in O(N \log N)$
- $\log N! \in \Omega(N \log N)$
- Both
- Neither
- Not sure

Algorithm Design Paradigms

Greedy Algorithms. Consider each option in order of lowest-cost.

- Prim's Algorithm.
- Kruskal's Algorithm.
- Dijkstra's Algorithm.

Caveat. Can lead to suboptimal solutions.

Dijkstra's algorithm on negative edge weighted graphs.

Divide-and-Conquer Algorithms. Solve two or more subproblems recursively, and then combine the results.

- Merge sort.
- Quicksort.

Prototypical usage. Turn brute-force N^2 runtime algorithm into N log N algorithm.

Algorithm Design Process

Hypothesize. How do invariants affect the behavior for each operation?

Identify. What strategies have we used before? What examples can we apply?

Plan. Propose a new way from findings.

Analyze. Does the plan do the job? What are potential problems with the plan?

Create. Implement the plan.

Evaluate. Check implemented plan.

Find a lower and upper bound. Define a slow but totally correct solution. Build a mental model: identify key properties.

Consider each algorithm that you know. Which ones might work? How do the existing algorithms break down?

Apply an algorithm design idea. Perform a reduction: transform input and output. Or modify the data structures used.

Use an algorithm design paradigm.

Q Counting Inversions

Given a permutation of length N, count the number of inversions.

0	2	3	1	4	5	7	6

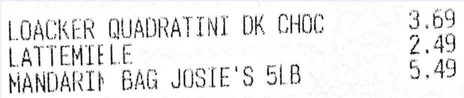
3 inversions: **2–1**, **3–1**, **7–6**

Lower bound? Upper bound? Desired runtime? Algorithm paradigm?

Q Optical Character Recognition

Suppose we're building an **optical character recognition** system.

We want to separate lines of text. There is some white space between the lines but problems like noise and the tilt of the page makes it hard to find.



How can we do line segmentation?